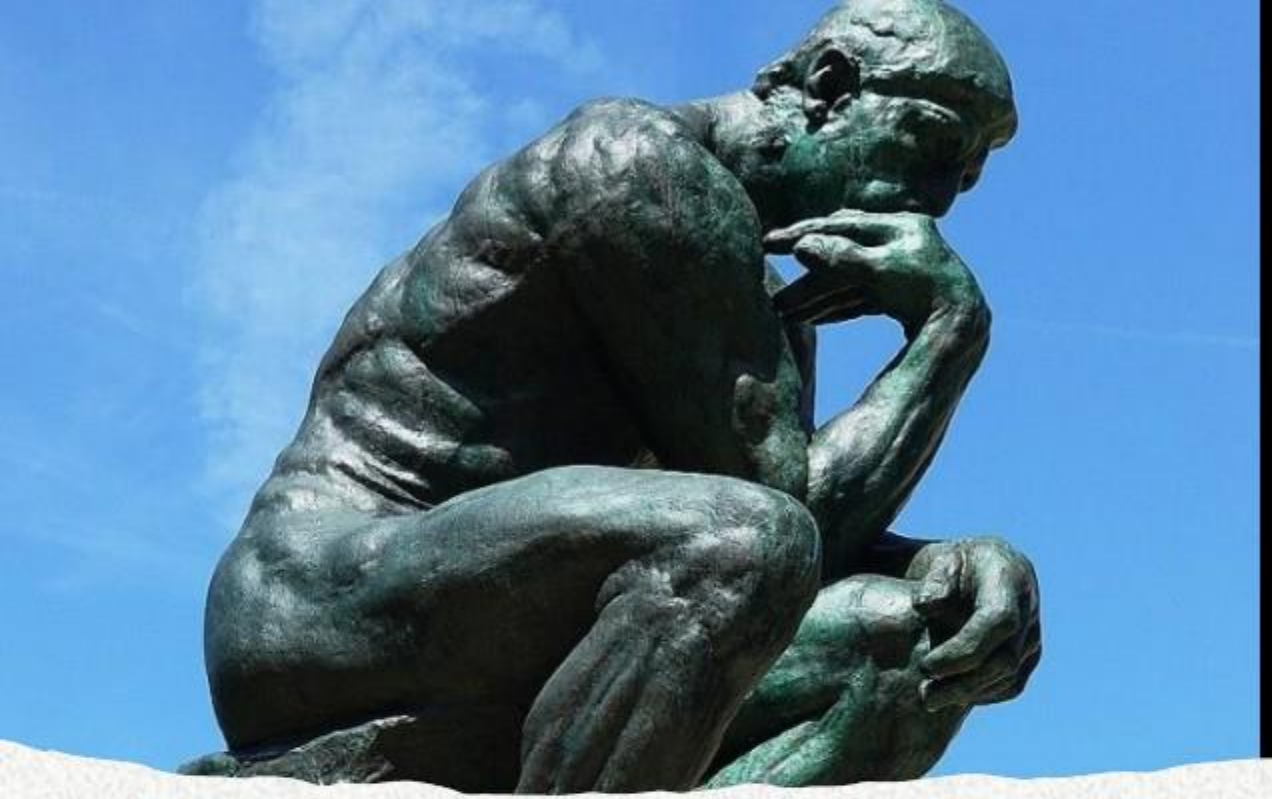
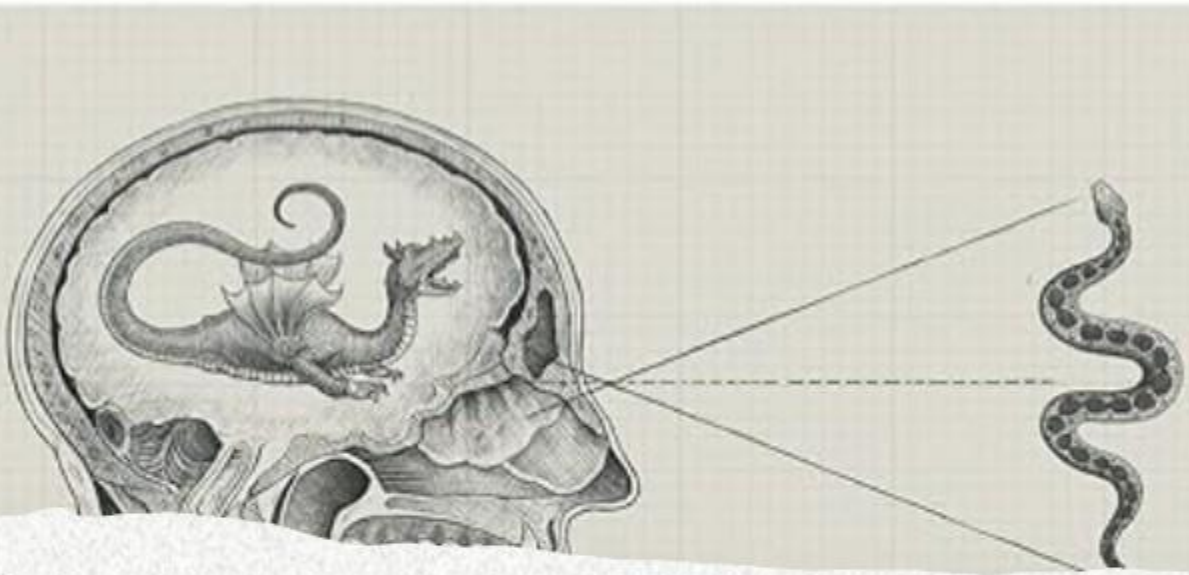


LOGIC

The Fundamentals of Thinking Well



Come Let Us Reason Together: Learning To Love God With All Thy Mind

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Hypothetical Syllogisms

Come Let Us Reason Together: Learning To Love God With All Thy Mind

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Hypothetical Syllogisms

- A **hypothetical** is a statement that affirms an outcome based on a condition. It has the form *If P then Q*.
- A **pure hypothetical syllogism** is an argument that uses only hypothetical statements. This form of argument employs hypotheticals, as follows:
  - If P then Q.
  - If Q then R.
  - Therefore, if P then R.
- We can use the symbol ⊃ for if. . . then. When we do, the whole argument is symbolized like this:
  - P⊃Q
  - Q⊃R
  - ∴P⊃R
- This is a valid argument which, in fact, could be translated into an AAA-1 categorical syllogism.

Hypothetical Syllogisms

- Here is an example of a valid, purely hypothetical syllogism:
  - If I study, then I will get good grades.
  - If I get good grades, then my parents will be pleased.
  - Therefore, If I study then my parents will be pleased.
- We see that the hypothetical statements combine two categorical statements into one new *if* . . . *then* statement.
- The categorical statement after the *if* is called the **antecedent**, usually abbreviated *P*.
- The statement after the *then* is called the **consequent**, abbreviated *Q*.
- The antecedent of the above conclusion is *I study*, and the consequent is *My parents will be pleased*.

Hypothetical Syllogisms

- Pure hypothetical syllogisms can also be invalid. Consider this argument:
  - If you are a woman, then you are a human.
  - If you are a man, then you are a human.
  - Therefore, if you are a woman, then you are a man.
- This argument follows this form.
  - If P then Q $P \supset Q$
  - If R then Q $R \supset Q$
  - Therefore, if P then R $\therefore P \supset R$
- Some syllogisms combine hypothetical and categorical statements. These are called **mixed hypothetical syllogisms**. We will consider two valid forms and two invalid forms.

Hypothetical Syllogisms

- The first form we will consider is called **modus ponens**. It looks like this:
  - If P then Q
  - P
  - Therefore, Q
- If we put terms from real life into the argument, we could obtain this:
  - If I study, then I will get good grades.
  - I study.
  - Therefore, I will get good grades.
- You can see that the first statement is a hypothetical statement, and the second is a categorical statement. If this is expressed with symbols only, we can clearly see the form of **modus pones**.
  - $P \supset Q$
  - P
  - $\therefore Q$

Hypothetical Syllogisms

- A second type of argument is called **modus tollens**. The form of argument is:
  - If P then Q
  - Not Q
  - Therefore, not P
- With the same real terms we used above, the argument would be:
  - If I study, then I will get good grades.
  - I did not get good grades.
  - Therefore, I did not study.
- In symbols, modus tollens would thus be:
  - $P \supset Q$
  - $\sim Q$
  - $\therefore \sim P$

Hypothetical Syllogisms

- There are also two fallacies that take a similar form to the arguments presented above. The first is the fallacy of **affirming the consequent**, so named because the second premise affirms the consequent of the hypothetical statement. This is how it looks:
  - If P then Q $P \supset Q$
  - Q Q
  - Therefore, P $\therefore P$
- With terms from the real world inserted, we see a form of invalid argument which is all too familiar.
  - If I study, then I will get good grades.
  - I got good grades.
  - Therefore, I studied.
- This is what is called a **non sequitur** [“it does not follow”], meaning “it does not follow.” The student may have gotten good grades some other way—cheating, for example.
- The initial statement does not say that studying is the only way to get good grades.

Hypothetical Syllogisms

- The other fallacy is called **denying the antecedent**, because the antecedent of the hypothetical statements is denied in the second premise.
  - If P then Q $P \supset Q$
  - Not P $\sim P$
  - Therefore, not Q $\therefore \sim Q$
- A real-life example is
  - If I study, then I will get good grades.
  - I did not study.
  - Therefore, I will not get good grades.
- This is a *non sequitur*. You might not study but still luck out and get good grades.

Valid	*Modus Ponens* If P then Q P ∴Q	*Modus Tollens* If P then Q ~Q ∴~P
Invalid	*Affirming the Consequent* If P then Q Q ∴P	*Denying the Antecedent* If P then Q ~P ∴~Q

Summary

- Hypothetical syllogisms are very common arguments in normal English. Pure hypothetical syllogisms employ only *if* . . . *then* statements.
- Mixed hypothetical syllogisms employ hypotheticals and categoricals.
- There are two valid forms of mixed hypothetical syllogisms: modus ponens and modus tollens.
- There are two invalid forms: affirming the consequent and denying the antecedent.